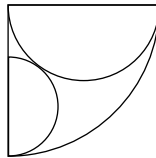
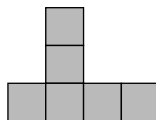


19. The diagram shows a quadrant of radius 2, and two touching semicircles. The larger semicircle has radius 1. What is the radius of the smaller semicircle?



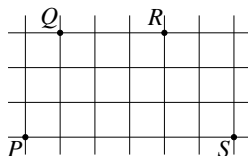
- A $\frac{\pi}{6}$ B $\frac{\sqrt{3}}{2}$ C $\frac{1}{2}$ D $\frac{1}{\sqrt{3}}$ E $\frac{2}{3}$

20. The diagram shows six squares with sides of length 2 placed edge-to-edge. What is the radius of the smallest circle containing all six squares?



- A $2\sqrt{5}$ B $2\sqrt{6}$ C 5 D $\sqrt{26}$ E $2\sqrt{7}$

21. Fiona wants to draw a 2-dimensional shape whose perimeter passes through all of the points P , Q , R and S on the grid of squares shown. Which of the following can she draw?



- (i) A circle (ii) An equilateral triangle
(iii) A square

- A only (i) and (ii) B only (ii) and (iii) C only (i) and (iii)
D all of (i), (ii) and (iii) E none of (i), (ii) and (iii)

22. A bag contains m blue and n yellow marbles. One marble is selected at random from the bag and its colour is noted. It is then returned to the bag along with k other marbles of the same colour. A second marble is now selected at random from the bag. What is the probability that the second marble is blue?

- A $\frac{m}{m+n}$ B $\frac{n}{m+n}$ C $\frac{m}{m+n+k}$ D $\frac{m+k}{m+n+k}$ E $\frac{m+n}{m+n+k}$

23. Which of the following have no real solutions?

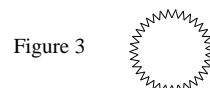
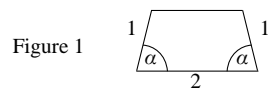
- (i) $2x < 2^x < x^2$ (ii) $x^2 < 2x < 2^x$ (iii) $2^x < x^2 < 2x$
(iv) $x^2 < 2^x < 2x$ (v) $2^x < 2x < x^2$ (vi) $2x < x^2 < 2^x$

- A (i) and (iii) B (i) and (iv) C (ii) and (iv)
D (ii) and (v) E (iii) and (v)

24. Which of the following is smallest?

- A $10 - 3\sqrt{11}$ B $8 - 3\sqrt{7}$ C $5 - 2\sqrt{6}$ D $9 - 4\sqrt{5}$ E $7 - 4\sqrt{3}$

25. Figure 1 shows a tile in the form of a trapezium, where $\alpha = 83\frac{1}{3}^\circ$. Several copies of the tile are placed together to form a symmetrical pattern, part of which is shown in Figure 2. The outer border of the complete pattern is a regular 'star polygon'. Figure 3 shows an example of a regular 'star polygon'.



How many tiles are there in the complete pattern?

- A 48 B 54 C 60 D 66 E 72



UK SENIOR MATHEMATICAL CHALLENGE

Thursday 6 November 2014

Organised by the **United Kingdom Mathematics Trust**

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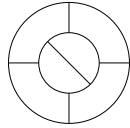
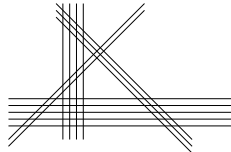
Institute
and Faculty
of Actuaries

RULES AND GUIDELINES (to be read before starting)

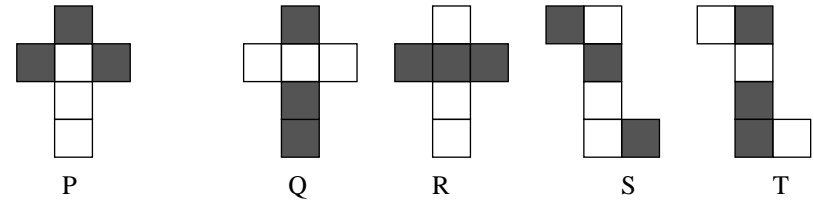
- Do not open the question paper until the invigilator tells you to do so.
- Use B or HB pencil only.** Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
- Time allowed: **90 minutes.**
No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
- The use of rough paper is allowed.
Calculators, measuring instruments and squared paper are forbidden.
- Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
- There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
- Scoring rules:** all candidates start out with 25 marks;
0 marks are awarded for each question left unanswered;
4 marks are awarded for each correct answer;
1 mark is deducted for each incorrect answer.
- Guessing:** Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

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<http://www.ukmt.org.uk>

1. What is 98×102 ?
 A 200 B 9016 C 9996 D 998 E 99 996
2. The diagram shows 6 regions. Each of the regions is to be painted a single colour, so that no two regions sharing an edge have the same colour. What is the smallest number of colours required?
 A 2 B 3 C 4 D 5 E 6
- 
3. December 31st 1997 was a Wednesday. How many Wednesdays were there in 1997?
 A 12 B 51 C 52 D 53 E 365
4. After I had spent $\frac{1}{5}$ of my money and then spent $\frac{1}{4}$ of what was left, I had £15 remaining. How much did I start with?
 A £25 B £75 C £100 D £135 E £300
5. How many integers between 1 and 2014 are multiples of both 20 and 14?
 A 7 B 10 C 14 D 20 E 28
6. In the addition sum shown, each of the letters T , H , I and S represents a non-zero digit. What is $T + H + I + S$?
 A 34 B 22 C 15 D 9 E 7
- | | | | |
|-------|-----|-----|-----|
| T | H | I | S |
| + | | I | S |
| <hr/> | | | |
| 2 | 0 | 1 | 4 |
7. According to recent research, global sea levels could rise 36.8 cm by the year 2100 as a result of melting ice. Roughly how many millimetres is that per year?
 A 10 B 4 C 1 D 0.4 E 0.1
8. The diagram shows four sets of parallel lines, containing 2, 3, 4 and 5 lines respectively. How many points of intersection are there?
 A 54 B 63 C 71 D 95 E 196
- 
9. Which of the following is divisible by 9?
 A $10^{2014} + 5$ B $10^{2014} + 6$ C $10^{2014} + 7$ D $10^{2014} + 8$ E $10^{2014} + 9$
10. A rectangle has area 120 cm^2 and perimeter 46 cm. Which of the following is the length of each of the diagonals?
 A 15 cm B 16 cm C 17 cm D 18 cm E 19 cm
11. A Mersenne prime is a prime of the form $2^p - 1$, where p is also a prime. One of the following is **not** a Mersenne prime. Which one is it?
 A $2^2 - 1$ B $2^3 - 1$ C $2^5 - 1$ D $2^7 - 1$ E $2^{11} - 1$
12. Karen has three times the number of cherries that Lionel has, and twice the number of cherries that Michael has. Michael has seven more cherries than Lionel. How many cherries do Karen, Lionel and Michael have altogether?
 A 12 B 42 C 60 D 77 E 84

13. Each of the five nets P, Q, R, S and T is made from six squares. Both sides of each square have the same colour. Net P is folded to form a cube.

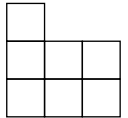


How many of the nets Q, R, S and T can be folded to produce a cube that looks the same as that produced by P?

- A 0 B 1 C 2 D 3 E 4
14. Given that $\frac{3x + y}{x - 3y} = -1$, what is the value of $\frac{x + 3y}{3x - y}$?
 A -1 B 2 C 4 D 5 E 7

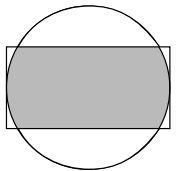
15. The figure shown alongside is made from seven small squares. Some of these squares are to be shaded so that:

- (i) at least two squares are shaded;
 (ii) two squares meeting along an edge or at a corner are not both shaded.



How many ways are there to do this?

- A 4 B 8 C 10 D 14 E 18
16. The diagram shows a rectangle measuring 6×12 and a circle. The two shorter sides of the rectangle are tangents to the circle. The circle and rectangle have the same centre. The region that lies inside both the rectangle and the circle is shaded. What is its area?



- A $12\pi + 18\sqrt{3}$ B $24\pi - 3\sqrt{3}$ C $18\pi - 8\sqrt{3}$
 D $18\pi + 12\sqrt{3}$ E $24\pi + 18\sqrt{3}$

17. An oil tanker is 100 km due north of a cruise liner. The tanker sails SE at a speed of 20 kilometres per hour and the liner sails NW at a speed of 10 kilometres per hour. What is the shortest distance between the two boats during the subsequent motion?

- A 100km B 80km C $50\sqrt{2}$ km D 60km E $33\frac{1}{3}$ km

18. Beatrix decorates the faces of a cube, whose edges have length 2. For each face, she either leaves it blank, or draws a single straight line on it. Every line drawn joins the midpoints of two edges, either opposite or adjacent, as shown.



What is the length of the longest unbroken line that Beatrix can draw on the cube?



- A 8 B $4 + 4\sqrt{2}$ C $6 + 3\sqrt{2}$ D $8 + 2\sqrt{2}$ E 12